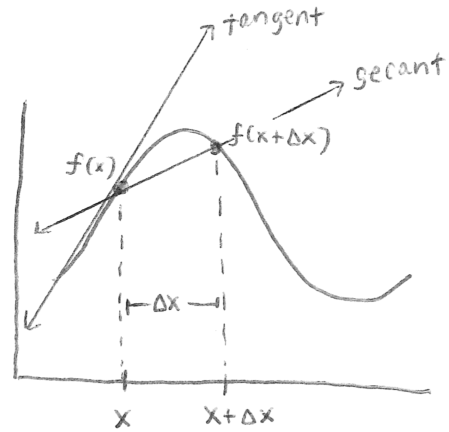
AP Calculus AB
Class Notes

Name Key



Definition Of the Derivative

"f prime of x"

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. $f(x) = x^2 + 6x - 2$

$$\lim_{h \to 0} \frac{(x+h)^2 + 6(x+h) - 2 - (x^2 + 6x - 2)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+6x} + 6h \cancel{-2} \cancel{-x^2} \cancel{-6x} \cancel{+2}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 + 6h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h + 6)}{\cancel{h}}$$

$$2x + (0) + 6$$

$$2x + 6$$

$$f'(x) = 2x + 6$$

2. $f(x) = 3x^2 + 7x + 3$

$$\lim_{h \to 0} \frac{3(x+h)^2 + 7(x+h) + 3 - (3x^2 + 7x + 3)}{h}$$

$$\lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 7x + 7h + 3 - 3x^2 - 7x - 3}{h}$$

$$\lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{+7x} + 7h \cancel{+3} \cancel{-3x^2} \cancel{-7x} \cancel{-3}}{h}$$

$$\lim_{h \to 0} \frac{6xh + 3h^2 + 7h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(6x + 3h + 7)}{\cancel{h}}$$

$$6x + 3(0) + 7$$

$$6x + 7$$

$$f'(x) = 6x + 7$$

3. $f(x) = \sqrt{x}$

$$\lim_{h\to 0} \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h} \frac{\left(\sqrt{x+h} + \sqrt{x}\right)}{\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$\lim_{h\to 0} \frac{\cancel{x}+h \cancel{-(x)}}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$\lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{1}{\sqrt{x+(0)} + \sqrt{x}}$$

$$\frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

4. $f(x) = \dfrac{1}{\sqrt{x}}$

$$\lim_{h\to 0} \frac{\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)}{h} \frac{\left(\sqrt{x}\left(\sqrt{x+h}\right)\right)}{\left(\sqrt{x}\left(\sqrt{x+h}\right)\right)}$$

$$\lim_{h\to 0} \frac{\left(\sqrt{x} - \sqrt{x+h}\right)}{h\left(\sqrt{x}\sqrt{x+h}\right)} \cdot \frac{\left(\sqrt{x} + \sqrt{x+h}\right)}{\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$\lim_{h\to 0} \frac{\cancel{x} + (x \mp h)}{h\left(\sqrt{x}\sqrt{x+h}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$\lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\left(\sqrt{x}\sqrt{x+h}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$\lim_{h\to 0} \frac{-1}{\left(\sqrt{x}\sqrt{x+h}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$\frac{-1}{\left(\sqrt{x} \cdot \sqrt{x+0}\right)\left(\sqrt{x} + \sqrt{x+0}\right)}$$

$$\frac{-1}{x\left(2\sqrt{x}\right)}$$

$$\frac{-1}{2x\sqrt{x}}$$

$$f'(x) = \frac{-1}{2x^{3/2}}$$

Alternative Definition of the Derivative

$$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

*used when you know the value "a" where you want the slope of the line tangent

1. Find $f'(2)$ if $f(x) = x^3$ $\quad f(2) = 8$

$\lim\limits_{x\to 2} \dfrac{x^3-8}{x-2}$

$\lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}}$

$(2)^2+2(2)+4$

$f'(2) = 12 \rightarrow$ slope of the line tangent to the curve $f(x)=x^3$ where $x=2$

equation of tangent line:
$y-8 = 12(x-2)$

2. Find $f'(9)$ if $f(x) = 2\sqrt{x}$ $\quad f(9) = 6$

$\lim\limits_{x\to 9} \dfrac{(2\sqrt{x}-6)}{(x-9)}\dfrac{(2\sqrt{x}+6)}{(2\sqrt{x}+6)}$

$\lim\limits_{x\to 9} \dfrac{4x-36}{(x-9)(2\sqrt{x}+6)}$

$\lim\limits_{x\to 9} \dfrac{4\cancel{(x-9)}}{\cancel{(x-9)}(2\sqrt{x}+6)}$

$\dfrac{4}{2(\sqrt{9})+6}$

$f'(9) = \dfrac{4}{12} = \dfrac{1}{3}$

equation of tangent line:
$y-6 = \frac{1}{3}(x-9)$

3. Find $f'(2)$ if $f(x) = x^2 + x$ $\quad f(2) = 6$

$\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x-2}$

$\lim\limits_{x\to 2} \dfrac{(x+3)\cancel{(x-2)}}{\cancel{x-2}}$

$f'(2) = 5$

equation of tangent line:
$y-6 = 5(x-2)$